

Cellular automata demonstrates synergy between deterministic chaos and emergent order

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Abstract. The border of order–edge of chaos has been a contentious concept since Mitchell et al failed to confirm Packard’s earlier results re Langton’s λ . New software has permitted study of long iterations from simple seeds under the Generations 345/3/6 cellular automata rule. Slowly growing chaotic cores infrequently form track/trail-laying engines which sometimes seed new chaotic nuclei at a distance, accelerating total growth. This productive synergy between order and chaos fits better with Wolfram’s original conception of Class 4 rules than with the common impression of chaos and order as polar opposites.

1 Introduction

In 1970 John Horton Conway selected and named the Game of Life (GoL), a two dimensional cellular automata (CA) which has dominated the field ever since (Gardner, 1970). William Poundstone (1985) used the emergence of complexity in GoL to counterpoint an information-theoretic approach to the emergent complexity of the natural world. MCell/MJCell author Mirek Wojtowicz (2000) noted the Generations 345/3/6 CA and named it “LivingOnTheEdge” (LOTE), commenting only: “In this very chaotic rule it’s hard to tell if patterns will survive or die out.” There is no evidence that the evolution of patterns under LOTE had been studied deeply before late 2008 when support for Generations rules was added in version 2.0 of GoL and general CA software Golly.

In 1983 Stephen Wolfram surveyed 256 simple one dimensional (1D) CA and proposed a classification scheme with four categories, his “Class 4” being those which develop “complex localized subpatterns, some of them long-lasting.” (Dewdney, 1985) In the early 1990s at The Santa Fe Institute, Chris Langton proposed a dimensionless number λ to locate the boundary between order and chaos (1991), a measure Norm Packard initially verified and then Mitchell, Crutchfield and Hraber failed to verify (1993). Wolfram’s (2002) in his magnum opus, *A New Kind of Science* (NKS), proposed a “Principle of Computational Equivalence” which further smudged the idea of a sharp divide between order and chaos.

In 1983, inspired by Ed Fredkin’s trivial solution to reproduction of patterns in CA, this author generalised Fredkin’s solution to develop Pattern Breeder (Dewdney, 1986). The author has studied GoL as new software permitted over

the intervening years under two simple constraints: (i) confined to narrow cylindrical universes (Life in a Tube) and (ii) the erosion of persistent tiled GoL patterns (agars) following a one cell disturbance. Following NKS, the author studied finite 1D system that emerged frequently in narrow Tubes and a simple rule for the evolution of graph theoretic networks.

The seemingly endless supply of surprising emergent phenomena when running LOTE from simple seeds quickly made it the sixth and largest of this author's studies of discrete systems. That project has provided hard evidence that the boundary zone between order and chaos is better understood as being where both can operate, not just interchangeably but synergistically, rather than as any kind of dividing line. That emergent synergy is the subject of this paper.

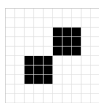


Fig. 1. Original seed of “The Wild”.

This study of LOTE started out focusing on evolution from one particular simple seed (Fig. 1). interrupted by quick surveys of simple collisions etc. to help explain many emergent phenomena post hoc. While discovery of three forms of track/trail laying during the first days of running that seed encouraged persistence, it was a simple clean break on one trail (Fig. 2) which defied imagination and underscored the significance of Generations 345/3/6. After the growth of the pattern generated from that first symmetric seed encountered system capacity constraints a common asymmetric seed was studied in comparable depth. With a second computer dedicated to pushing those seeds to new levels it became possible to concurrently survey a wider range of seeds to moderate depth.

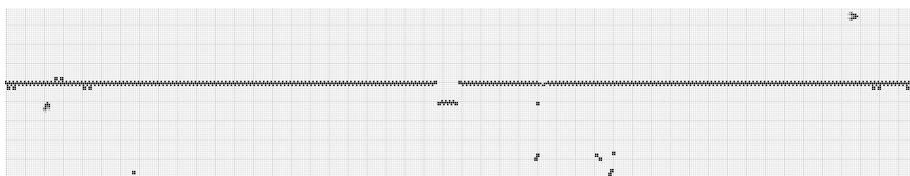


Fig. 2. First seen aftermath, at iteration 175600 of The Wild, of the unbelievably neat “Hanu” mechanism which moves 11 cells of track 10 cells to the side.

Terminology: In this paper ‘iteration’ is used rather than ‘generation’ to identify step/CA time count to avoid confusion with the name of the ‘Generations’

family of rules. With no diagonally moving spaceships found, LOTE's common orthogonal spaceships are generally referred to as 'ships'.

2 Emergent patterns in LOTE

Mirek Wojtowicz (2000) described a new family of CA rules:

The 'Generations' game rules are very close to those from Life, with one addition: the cells' history. Cells that would simply die in Life are only getting older in Generations. They cannot give birth to new cells, but they occupy the space of the lattice, thus changing the rules radically.

Generations 345/3/6 means:

- live cells with 3, 4 or 5 neighbours stay alive, otherwise they start dying;
- empty cells with exactly 3 live neighbours come alive;
- there are a total of 6 cell states: empty, live and 4 dying;
- the next state of empty and live cells depends only on the count of live cells in the cell's Moore neighbourhood: its 8 surrounding cells;
- the next state of a dying cell depends only on its current state: it does not look at nor is it looked at by its neighbours

Patterns in this paper show empty cells as white, live cells as black and dying cells in lightening shades of grey (Fig.3).

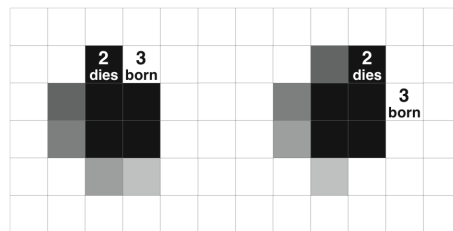


Fig. 3. Consecutive phases of a ubiquitous period 8 cyclic pattern noting the live neighbour count that will cause cells to come alive or start dying.

Like GoL, LOTE has a range of small stable and cyclic patterns which occur at generally consistent frequencies (Fig.4).

This paper is solidly on the naturalist side of the divide into naturalists and engineers Poundstone (1985, p.47) finds within those studying GoL:

There are at least two ways of looking at Life. One is the naturalist approach, in which you are interested mainly in seeing what objects occur naturally—what order arises from chaos. The other is the engineer approach, in which you try to construct objects that do complicated, clever, or surprising things.

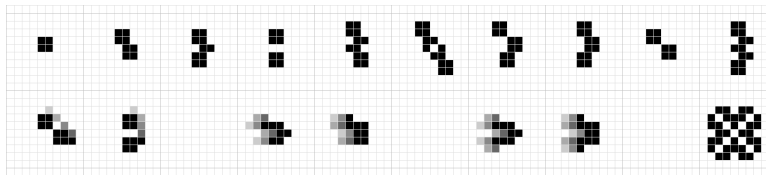


Fig. 4. Ten common stable shapes, period 12 and 14 cyclic patterns, both phases of the two common ships and the stable “x” pattern often seen from Golly’s random fill.

Poundstone then illustrates how an irreversible CA produces natural objects in attractor basins. One of Conway’s original criteria for selecting GoL was that patterns did not grow indefinitely and he offered a reward for anybody who could find one which did. None appeared naturally, but glider guns, puffer trains and switch engines were soon constructed by combining promising unstable patterns with convenient stabilising patterns. Golly is optimised for this kind of engineering with 4 decades more ingenuity and computing power.

In LOTE, some small seeds grow without limit, passing a threshold of viability and then following a broadly consistent, though chaotic, growth pattern until a secondary growth centre is formed, after which diversity becomes the norm. Vastly more small patterns either disappear or stabilise, so overall growth from viable seeds remains moderate. Beyond that gradual chaotic growth, LOTE naturally produces a wide variety of what are strictly puffer trains.

3 Finding viable seeds

The original seed run to great depth, ‘The Wild’ (Fig.1), was a fortuitous mistake. A visually pleasing growing pattern with double diagonal symmetry arises through a variety of ship-ship collisions, ship-common object collisions and within active chaotic areas. A recognisable form early in its evolution is similar to The Wild seed, except with the central cell of each lobe in the first dying state. In the default colours of MJCell and Golly that difference was not conspicuous to a then Generations rules novice.

After the first days running The Wild revealed 3 track/trail/puffer forms, it made sense to survey all collision geometries between common ships. As with GoL, the most frequent result was mutual annihilation, followed by one, two or a small cluster of common shapes. But some seeds stayed active a lot longer, up to 7850 iterations for one cyclically symmetric seed to reach a final form including 24 outbound ships in the four primary directions. And there were 4 distinct seeds that keep growing:

- the ubiquitous double diagonally symmetric seed (ddsym) produced by both head on and orthogonal collisions;
- another 180° cyclically symmetric seed;

- one head on asymmetric collision saddled with the forced acronym ‘Whoa’ for “Wild head on asym” and since run for more iterations than The Wild because its lack of symmetry quadruples opportunities for observation;
- one orthogonal asymmetric collision which, even more than Whoa, forms early remote growth centres.

Collisions between common ships and common stable and cyclic forms show a similar rate of production of viable seeds, as do collisions between two less common ships and the commonest ships and objects. A small number of other seeds have been found by detailed observation within large patterns, a process heavily biased towards identifying symmetric seeds.

While the 180° cyclically symmetric seed does not develop differently to typical asymmetric seeds, save for producing half as many distinct results, other naturally occurring symmetric seeds show distinctive overall growth patterns:

- diagonally symmetric seeds grow significantly faster but still chaotically along the diagonal(s), even before sustained $(\frac{1}{4}, \frac{1}{4})$ speed diagonal growth is established by ddsym production from symmetric puffer rakes;
- bilaterally symmetric seeds with a double row of cells on their axis produce trail-laying engines on the axis with growth along the axis accelerated by bursts of often spectacular trail-consuming growth.

Within a larger pattern, new seeds formed by ship collisions etc. only follow the development path of that same seed in isolation until an effectively random ship arrival disturbs their development. Likewise, random ship arrivals disturb settled clusters within ship streams, notably reactivation of the tail of a common puffer trail. All such interactions are computationally irreducible and thus unpredictable at the detail level, even while most follow a recognisable pattern.

4 Core and outflow dynamics

Viable seeds develop a chaotic core that is a mixture of temporarily stable and often quite large maze-like structures with active edges and shoals of small stable or cyclic patterns. Those cores become the base of narrow triangular beams of ships heading NSEW at one cell every two iterations.

The growth of active core areas has so far defied detailed analysis with no indicators of regularity. Viewed from a higher level, the growth appears to be largely the product of localised bursts similar to the growth of isolated viable seeds. Such growth bursts expire leaving a shoal dominated by common stable and cyclic patterns at near critical density plus some ships adding to the outbound stream. (As ships are equally likely to be formed in any orientations, those not outbound interact within the core, sometimes reactivating settled shoals.)

Save for any preexisting global seed symmetry, the early shape of a growing core is highly irregular with often convex promontaries formed by those bursts separated by concave bays rounded by infill. As an asymmetric core grows it initially tends circular, through the kind of aggregation that makes cauliflowers circular. Then as space for orthogonal collisions in the corners grows, the core tends more square.

5 Engines, tracks and trails

The first indication that Generations 345/3/6 might be particularly interesting was the emergence in The Wild of an “engine”, the head of which moved and looked like a pair of ships, but left behind a “track” of the same form as already familiar stable patterns. Rerunning the initial formation of this pattern revealed a complicated emergence (Fig.5), before it settled into alternate modes:

- with a block anchoring the track it lengthened at ship speed ($\frac{1}{2}$);
- after a following ship removed the anchor block, the track eroded by 6 cells per period 10 cycle, shrinking at net $\frac{1}{10}$ speed.



Fig. 5. “Alpha”, the first track laying engine seen in The Wild, at iteration 56,284 still with the tagalongs it picked up soon after its emergence.

Between the $\frac{1}{2}$ speed growth, the $\frac{1}{10}$ speed shrinkage and the effectively random arrival of deblocking ships, those alternating modes rapidly lengthened, at least until something even less expected occurred.

When eroding track reaches a common form of engine it has an equal chance of reblocking or of converting to a period 192 puffer. Each cycle the puffer emits two ships orthogonally to the direction of travel of ships in the stream. Interaction dynamics ensure that almost all of the rakes thereby formed eventually penetrate the full width of the stream. The first puff stabilises into a larger shoal than those that repeat every 96 cells along the rest of the length of the growing puffer trail. That first puff interacts with following ships until it reignites as a new island of chaotic growth which could not avoid being tagged “Honalee”.

As that first Honalee starts to grow in The Wild, a second engine, “Homer”, forms further off axis on the active core face, a different class of engine which leaves a trail of blocks, one per 4 cells travelled/period 8 cycle. The underlying relatedness of engine forms is made clear when an interaction with a ship from growing Honalee converts Homer to laying track the way Alpha started out. That track continued to be impacted by crossflow ships from Honalee, eventually producing that one still more improbable reaction (Fig.2).

Ships from Honalee meet ships from the core head on and orthogonally, seeding new growth which is absorbed into a growing chaotic area. This can send live cell count growth exponential and only very slowly drop back towards the quadratic limit that geometry dictates. An initially period 56 engine also appears in The Wild but rarely, its trail taking two forms differentiated by a single variation 180 cells behind the head: (i) a rakeless periodic trail; and (ii) a period doubling form in which the periodic region grows without escaping contact

with a trailing chaotic region, producing a narrow active wedge flanked by an expanding delta wing of rakes. In Whoa, the rakes of a “naked” Delta visually dominate from soon after its formation until long after its eventual disruption.

With a second computer dedicated to further iterating those original seed choices, studying a wider range of seeds in less depth concurrently became an option. By 100,000 iterations almost all viable seeds examined have also produced several track or trail laying engines. Without some additional structure, the population of live plus dying cells at 100,000 is close to 4 million.

Under a range of circumstances, tracks and trails can cause new active centres to form out along these beams where interaction with a statistical flux of following ships produces rapid expansion of the active area. These new active centres produce new tracks and trails, albeit at a similar slow rate, eventually producing a wild divergence of active core and side beam shapes. Such accelerated growth and bulk shape variation occurs in half those so far run to 100,000.

6 Orderly, resilient structures spreading chaos

Tracks and trails each exhibit their own forms of resilience in a range of synergistic interactions which are dependent on the existence of the infrastructure provided by the tracks and trails themselves, phenomena so unexpected from previous experience with cellular automata that specific terms needed to be coined to identify them.

This paper focuses on the interactions of two distinct classes of orderly patterns which enable new chaotic islands to be established far from the core where those orderly patterns emerged. Another class of processes which form Delta-like narrow chaotic wedges at engine/ship speed is known to also occur under other rules and is not examined here.

6.1 Track wreckage, tagalongs

When either of the common ships impacts the side of an unmodified track, there is a fifty-fifty chance that it will leave a pair of blocks attached to the side of the track or leave the track unmarked. This critical difference is whether the leading cell of the ship is aligned with a live cell on the edge of the track or with a gap between such cells. As the origins of such ships are far from the origins of the track, these alignments are computationally irreducible.

Interactions when $\frac{3}{5}$ track erosion reaches first order wreckage provides a foretaste of complex combinatorics. There are three equally likely outcomes:

- the track reblocks quickly;
- the track reblocks with a nearby active pattern which takes a while to settle;
- the track reblocks with a nearby active pattern which eventually reacts with the new block resuming $\frac{3}{5}$ erosion delayed.

When a second ship impacts first order wreckage, there are 9 outcomes including the wreckage being cleared, being left unchanged and those shown (Fig.6).

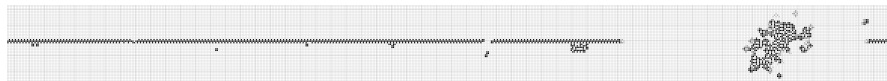


Fig. 6. Primary wreckage (left) resulting from a ship reaching a track, the 6 settled and one viable active patterns of secondary wreckage.

Interactions of a third ship with secondary wreckage or with a close pair of first order wreckage produces a range of outcomes, as does interaction between wreckage and eroding tagalong tails.

A “tagalong” head takes 3 iterations to move 2 cells along preexisting track, forming new track displaced by 3 cells from the original. While the forward mechanism is the same, there are three distinct outcomes:

- the original trail is left intact and a section of symmetric double track formed, the shorter side eroding at $\frac{1}{2}$ speed in a period 4 cycle;
- the original trail is stripped of the row of cells closest to the new track leaving a gap, tails of stripped tracks eroding 4 cells every 7 iterations;
- the original trail is completely removed, also at $\frac{2}{3}$ speed, seeming to move the trail sideways, a mechanism formed by one deblocking geometry.

Tagalongs encounter various instabilities, a common outcome being the addition of further tagalongs, eventually leading to more spectacular instabilities and often with tagalong heads escaping to seed more activity down the track.

When many track laying ships first form, the track does not cleanly break free from surrounding active areas so they are formed with tagalongs (e.g. Fig.5) which soon reach the head. Tracks can also pick up tagalongs after resisting later encroachment of active areas. Tertiary wreckage includes production of tagalongs as do some engine disruption geometries.

Tagalongs eroding from both ends usually leave the other track unmarked, except when period 4 erosion at both ends of a gapless tagalong is in sync. Then it matters whether the centre of the erosion is at a live cell or a gap. In one case it produces a symmetric ship that leaves the base track unmarked. In the other case it does something far too clever to have been anticipated (Fig.2).

While tagalongs reaching engines generally produce disordered active areas, long runs of multi-tagalong erosion can produce a range of periodic trails. Others can produce very complicated periodic structure (Fig. 7).

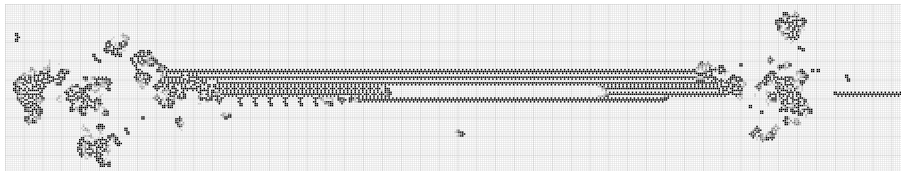


Fig. 7. Braided edge and periodic infill forming at iteration 233,000 of Whoa.

6.2 Block trail shifters, pythons, gobblers

Block trails are modified by ships, so must be within a ship stream to support complex reactions. “Shifter” is a period 10 process which moves 4 cells across a block and gap while reforming the block one cell back. At $\frac{2}{5}$ speed, shifters are slower than anything else moving regularly and they can repair trail gaps.

One reaction with a ship from the side produces a ship which skims along the trail 2 cells off and which can react with a shifter depending on their phases. Such reactions can happen far along a block trail, leaving off-trail debris to be reactivated by following ships.

Two mechanisms run along block trails: (i) trails on a bilaterally symmetric axis are swallowed by “pythons” running at $\frac{4}{5}$ speed until they overtake a shifter, after which they retreat laying blocks back; and (ii) an asymmetric “gobbler” consumes trails leaving puffs of debris every 48 cells in a period 78 cycle.

While the head is different, the python body is like a gapless tagalong and likely to pick up additional tagalongs, but 2 at a time, forming an “anaconda” which can develop complicated internal erosion (Fig.8).

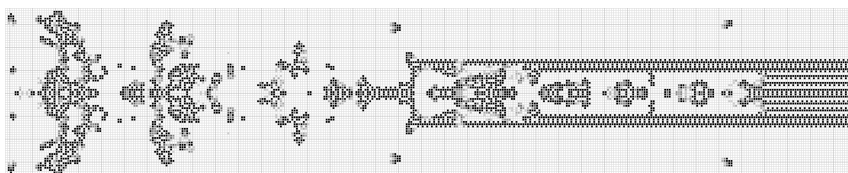


Fig. 8. Period 84 rake-producing erosion at iteration 20,960 of another seed.

The interaction combinatorics when a gobbler catches a shifter took a long time to comprehend. The relevant geometry repeats with period 42 within which there are 18 distinct processes, two of which remain active.

7 Increasing emergence through synergistic interaction of chaos and order

Langton’s λ parameter (1991) was intended to locate a phase transition between order and chaos. The experiments used to first confirm (Packard) and later discredit (Mitchell et al, 1993) λ used genetic algorithms to explore CA rule spaces far too large for exhaustive study. While it is now clear that there is no such phase transition, for a decade Mitchell et al’s response was over-interpreted, devaluing by association Packard’s earlier “Adaptation toward the edge of chaos” (1988). Wolfram (2002) too moved from an initially clear characterisation of “Class 4” as developing “complex localized subpatterns, some of them long-lasting” (Dewdney, 1985) to suggesting his “Principle of Computational Equivalence” might also encompass Class 3 rules that had been seen as entirely chaotic.

The results reported above from Generations 345/3/6 (LOTE) show not just that orderly structure can arise from deterministic chaos but that such order can synergistically accelerate the propagation of active chaos.

The rich and diverse results from LOTE ultimately depend on many minutiae which could never be predicted from a simple code like “345/3/6”, let alone λ . The minutiae of GoL presented by Poundstone (1985) show an approach that is needed even more so for LOTE. While accelerating the spread of LOTE’s distinctive form of chaotic core activity, the tracks and trails also preserve traces of prior activity, a prerequisite for higher functionality.

Alan Tennant has reported on other rules to Golly Test List. Generations 124567/34/6 rapidly produces a rich variety of ships and puffers plus a noteworthy wick stretcher from a trivial seed, all of which combine to accelerate the growth of a rapidly settling core by Delta-like processes. In a follow up conversation, he suggested a more targeted approach to identifying productive rules:

Turn up the last number (...) in the rule to get it more stable and less expanding or shrinking, you do lose some of the variation as a result and there is no one value that is best, there is impressive amounts of variation between all the possible values.

The self-perpetuating engines, their tracks and trails that emerge in The Wild and from other small seeds under LOTE each have a story, albeit one that can be rerun, undone and otherwise investigated with Golly. They emerge without design and have lives that feel almost natural, especially when viewed from the privileged perspective of a zoomable computer display that makes evident the near misses. More globally, there is pattern within no pattern (active core, ship flow) within pattern within no pattern, and that is just for starters.

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